Synthese
DOI 10.1007/s11229-014-0419-0

The relativity and universality of logic

Jean-Yves Beziau

Received: 17 August 2013 / Accepted: 7 February 2014

Abstract After recalling the distinction between logic as reasoning and logic as theory of reasoning, we first examine the question of relativity of logic arguing that the theory of reasoning as any other science is relative. In a second part we discuss the emergence of universal logic as a general theory of logical systems, making comparison with universal algebra and the project of *mathesis universalis*. In a third part we critically present three lines of research connected to universal logic: logical pluralism, non-classical logics and cognitive science.

Keywords Logic · Universality of logic · Relativity of logic · Universal logic · Universal algebra · Mathesis universalis · Characteristica universalis · Logical pluralism · Non-classical logics · Paraconsistent logic · Cognitive science

1 Two questions and a double meaning

Is logic relative? Is logic universal? These are fundamental questions regarding the very nature and meaning of logic. To properly answer these questions it is important to make the distinction between logic as *reasoning* and logic as the *theory of reasoning*. This is a distinction that is very easy to understand but most often not explicitly made. For details about this distinction see our recent paper "Logic is not logic" (Beziau 2010), where we chose to use the word "Logic" for reasoning and the word "logic" for the theory of reasoning. We will go on here with this distinction and the way to express it.

Dedicated to Istvan Németi for his 70th birthday.

J.-Y. Beziau (✉)
Department of Philosophy, Federal University of Rio de Janeiro, Brazilian Research Council, Brazilian Academy of Philosophy, Rio de Janeiro, Brazil
e-mail: beziau@gmail.com

Published online: 20 March 2014

2 The logical relativity of logic

2.1 Two extreme positions

Relativity of reasoning can be considered from the point of view of the subject or/and of the object. Extreme relativity would be for example to argue that Peter's way of reasoning about chocolate is different from Peter's way of reasoning about infinity and different from Lolita's way of reasoning about chocolate. This extreme relativism is not necessarily absurd. One may argue that reasoning varies according to sex, age and education. Imagine that Peter is a middle-aged mathematician and Lolita a young teenager. And also one may argue that reasoning varies according to the objects under examination—reasoning about chocolate is not necessarily the same thing as reasoning about Aleph 1. The Logic of transfinite is not the same as the Logic of chocolate.

Opposed to this extreme relativism is an extreme absolutism according to which human beings are all reasoning in the same way independently of the circumstances. This extreme absolutism is also not so absurd and in fact has been quite successfully promoted in the occidental civilization. It is related to science and democracy. It can be associated to Descartes claiming in the *Discourse on the method* (1637) that "le bon sens est la chose du monde la mieux partagée" ("Good sense is, of all things among men, the most equally distributed"), to the rationalism of *Siècle des Lumières* (*the Age of Enlightenment*) and to the *Déclaration des Droits de l'Homme et du Citoyen* (1789) later on shaped in a *Universal Declaration of Human Right* (1948).

Louis Rougier, one of the promoter of relativism in logic, was in fact consistently against democracy and rationalism, but he didn't present any very satisfactory alternative theory: in science he promoted conventionalism, in politics neo-liberalism. Rougier (1955) symbolically expressed the relativity of logic in the following way: "With the discovery of the conventional and relative character of logic, human spirit has burnt his last idol."[1]

It is worth also to note that Rougier was anti-Christian (see e.g. Rougier 1977), considering that Christianism was opposed to science. This can be interpreted against the universalism of Christianism about which we will come back later. It is interesting to emphasize that the French expression *Siècle des Lumières* is plural. This choice was consciously operated to make the distinction between the many lights of scientific knowledge by opposition to one absolute light of God. *Lumières* means we have different ways to look at reality; this can easily be turned into a relativism of different viewpoints, but we are still far from the conventionalism shared by Rougier and Carnap.

2.2 Science in transformation

Contemporary logic with its chaotic multiplicity of systems of logic seems to promote extreme relativism, but in the middle of this chaos there are still some individuals

[1] Rougier was close to the Vienna Circle, helping to promote it. His paper " The relativity of logic " (1941) has been recently re-edited in the *Anthology of Universal Logic* with comments by Mathieu Marion (2012).

claiming that a given logic is the true one, in the sense that a particular system of logic is a true description of the true way of reasoning similarly to what Kant was claiming about Aristotle's logic more than two centuries ago in the preface of the second edition of the *Critique of Pure Reason* (1787).

We can forgive Kant but it is more difficult to be so indulgent with such position nowadays. At the time of Newton's physics, a model for modern science, there was the idea of a scientific theory perfectly describing an objective reality lying in an absolute time and space. Since then quantum physics has seriously challenged the objectivity of reality, but also the idea of absoluteness of scientific theories has also vanished.

This vanishing is quite independent of the question of objectivity of reality and is rather connected to the astonishing development of science these last 150 years, showing science in perpetual transformation. Nowadays it seems quite normal to defend at the same time the idea of an objective reality and the relativity of scientific theories. This is perhaps due to the emergence of evolution, not just as a scientific theory to explain biological phenomena, but as a *Weltanschauung*. Everything is evolving: plants, animals and ideas. Piaget (1950) has developed a genetic epistemology, establishing a parallel between the development of intelligence of an individual in the course of his life and the development of science in the course of the life of humanity.

Relativity of a theory may be considered in relation to reality, not only according to the linear approximation of progress, following which we are getting closer and closer to reality (a kind of weak relativism, since in this perspective theories are less and less relative), but in a more logical way. Order between theories is not necessarily linear, it can be partial, one theory may be, in some sense, closer to reality than another one, and farther, in another sense. Moreover two theories can be contradictory, or incompatible—to speak in a less dramatic way. This kind of relativism nevertheless doesn't coincide with subjectivism: it is still possible to argue that science is objective in the sense that a scientific theory is not a personal idiosyncratic viewpoint but a rational proposal that everybody can understand, discuss and test.

On the other hand, if we consider that there is no objective reality, everything can become very confused, unless we stay very logical. It is possible to do so without going backward. Sokal and Bricmont (1997) have strongly criticized post-modernism, but what they have proposed is nothing else than a come back to the *Siècle des Lumières*. However travel through time is not (yet) possible, we have to face the present, and we are living at the time of quantum physics. Sokal (1996) in its hoax has promoted confusion because he has juxtaposed quotations by Bohr and Heisenberg rejecting objective reality with post-modernism nonsenses. Since he has proposed no other interpretations of quantum physics one may conclude that Bohr and Heisenberg's ideas have the same value as post-modern nonsenses. But surely what great physicists like Bohr and Heisenberg have said is philosophically very deep. The truth is that quantum physics seriously challenges the notion of objective reality.

Physicists and logicians have been working on that. Let us point out here the work of Jean-Louis Destouches and his wife Paulette Février. Destouches was the main student of Louis de Broglie and had a strong interest for logic. He proposed a generalization of logic in view of quantum physics extending it to what he called *quasi-formal logic* (see e.g. Destousches 1948)—a view of logic according to which we have to take into account the meaning of propositions. This is a framework allowing the development of

the three-valued logic of Février (1937), which gives a logical account of Heinsenberg's principle of indeterminacy.

Paulette Février was a close friend of Alfred Tarski (cf. Feferman and Feferman 2004) and together with Jean-Louis Destouches they were working towards logic as a general methodology of deductive science following the Polish school, promoting the development of the axiomatic method for all sciences, in particular for physics. Both were present at the congress organized by Patrick Suppes at the end of the 1950s in Berkeley about axiomatics in science in the spirit of Tarski's model theory (Henkin et al. 1959). According to model theory, a given theory may have different contradictory models (this does mean that a theory is contradictory). As it is known, Gödel (1949) has shown that Einstein's field equations of gravitation may have a model where time is circular, but there are models of this same theory where time is not circular. In a different perspective, I was myself led to develop the paraconsistent logic Z as a logical account of David Bohm's view about physics according to which we may have different viewpoints about the same reality (Beziau 2006a).

2.3 Logic in transformation

After 2000 years of immobility, the theory of reasoning, since Boole, like all other sciences, is in constant transformation. And a position of objective relativism in logic, according to which there is an objective reality of reasoning, but that any theory of it is relative seems quite natural. At some point some people had the idea that classical propositional logic and 1st order logic were a perfect description of our way of reasoning. But today there are many systems of logic and this seems as absurd as considering that syllogistic is the right description of reasoning.

However, there are still some people claiming that they have found the right logic. The confusion is connected with two problems: the double meaning of logic we have already mentioned and the duality descriptive/normative. Let us consider the case of intuitionistic logic. Some people defend that intuitionistic logic is the true reasoning in mathematics following Brouwer. But Brouwer was defending intuitionistic Logic as reasoning, and was against intuitionistic logic as a theory. Later on Heyting developed intuitionistic logic (see e.g., Moschovakis 2009).

First, it is important to note that there is not only one intuitionistic logic, but a whole variety of intuitionistic systems of logic. Even if someone thinks that intuitionistic Logic is the one true essence of mathematical reasoning, he may admit that there are various approximate theories of it. And there is another point: Brouwer claimed that intuitionistic Logic is the way we should reason, we should not follow classical Logic (see van Stigt 1990). There is something normative in this perspective that we don't have in physics, since it would be difficult to say that the universe should behave in such or such way.

A normative approach can be viewed as a kind of moralist perspective. Against this view, Carnap (1934) claimed that "in logic there are no morals". This is linked to the conventionalist perspective of his friend Rougier.

Independently of this conventionalist approach, we may argue that reasoning itself may change by new practices and/or by reflecting about it. Mathematics can be viewed

as the transformation of reasoning by practice and mathematical logic as a transformation of mathematical reasoning by reflection. It makes sense to say that Cantor's diagonal argument, as well as Gödel's argument used in his incompleteness theorem, are new ways of reasoning.

3 The road to logica universalis

3.1 Universal logic versus universal logic

For 20 years now, I have promoted the expression "universal logic" to develop logical research (for a general understanding of this perspective, see Beziau 2006b). This expression is highly ambiguous because it can mean two opposite things: a general theory of logical systems, a universal system of logic. We can extend the difference between *logic* and *Logic* to this double case speaking respectively about *Universal logic* and *Universal Logic*. I will explain here why I decided to use this expression despite this strong ambiguity.

The idea to develop a general theory of logical systems was the main concern of my PhD (1990–1995—Dept of Mathematics, University Paris 7—Denis Diderot, supervised by Daniel Andler). As I have recalled elsewhere (Beziau 2001), the starting point was the theory of valuations of Newton da Costa, as a continuation of my Master thesis on paraconsistent systems developed by this Brazilian logician. The idea of da Costa was to use bivaluations in a rather abstract way to develop a general approach for the study of logical systems (see Beziau 2012a). In 1991–1992, I went to São Paulo to work with him on this topic and the result was two joint papers (see da Costa and Beziau 1994a,b).

At the same period, I started to develop the idea of a more general theory of logics based on the idea of structure in the sense of modern mathematics. My idea was to consider logical structures as abstract structures at the same level as Bourbaki's mother structures (Bourbaki 1948). I was thus naturally led to use the expression *abstract logic* by analogy to *abstract algebra*. I liked the word *abstract* because it reflects a typical feature of human intelligence strongly connected with mathematics. Moreover I thought it was interesting to use the word *abstract* in opposition to the word *formal*. I had the idea that the expression *formal logic* (originally due to Kant) was much confused and that promoting the expression *abstract logic* would help to clarify things, giving a good name to a new way of conceiving logic, a new shape of logic quite different from logic as conceived at the turn of the twentieth century. In 1992 I decided then to choose as a provisional title of my PhD: *From formal to abstract logic*.

However in 1993 when I was in Poland, I decided to shift to *universal logic*, a shift related to polish logic. In 1992, still in Brazil, I discovered the works by Suszko (1973), who was already using the expression *abstract logic*. In one sense my approach was very close to Suszko's one, in continuity with Tarski's approach to the consequence operator (Tarski 1928): Suszko was considering logics as abstract mathematical structures. But Suszko's view was that abstract logic was part of universal algebra following a line of thought according to which universal algebra is a general theory of mathematical structures. Influenced by Bourbaki's architecture of mathematics I didn't share this point of view. For me it was important to emphasize the difference between

logic and algebra. I was against the reduction of logic to algebra which seems to me absurd both from a philosophical (a "sufism") and a mathematical view point (I had been working on some paraconsistent systems which are not algebraizable). Since the expression *abstract logic* had been connected to the spirit of the reduction/inclusion of logic to algebra, I thought it would be better not to use it. At this same moment another expression clearly appeared to me: *universal logic*.

While in Poland learning more about Suszko's approach, I was trying to have a better understanding of universal algebra and read several fundamental papers related to its history. At first, to shift from *abstract logic* to *universal logic* to avoid a too tight connection between logic and algebra may seem absurd since the expression *universal logic* is strongly connected with the expression *universal algebra*. And, in fact, I decided to use it because of this connection. What I liked about the idea of universal algebra as promoted by Garrett Birkhoff (1946) was the idea to throw away axioms, a strategy I characterized later on as *axiomatic emptiness* (Beziau 2010a). My idea was to develop a general theory of logics based on this same spirit of emptiness, throwing away Tarski's axioms for the consequence operator, but starting with a different primitive notion than in algebra. I started to lecture and write about that (Beziau 1994). Many years later, when I had already organized several world congresses on universal logic and launched the journal *Logica Universalis* and the book series *Studies in Universal Logic*, Luis Estrada pointed me that Melvin Fitting wrote in (1992): "Perhaps someday there will be a subject, universal logic, as there is already universal algebra". This was in a preface of the *Journal of Logic and Computation*. So the prophecy of Fitting came true, and in fact there is now a corner of universal logic in the JLC (that Dov Gabbay invited me to edit after his participation at the first World Congress on Universal Logic in Montreux in 2005).

3.2 Universal algebra and universal logic

It is difficult to know exactly why the expression "universal algebra" was chosen by Sylvester (1884) in the nineteenth century (about that see Riche 2007). But it seems that in the first period of universal algebra, the pre-Birkhoff period, what was predominant was to look for some universal laws, axioms or principles, not only for all algebras but for all mathematics. From this point of view we understand why Whitehead was considering his work with Bertrand Russell *Principia Mathematica* (Whitehead and Russell) as a continuation of his book *Universal Algebra* (1898). But Birkhoff broke this direction or better transformed it, turning the axiomatical into the conceptual. This transformation is indeed a metamorphosis, a flight into abstraction. What Birkhoff developed is not an axiomatic basis but a conceptual basis resting on abstract algebras as structures and morphisms. Birkhoff's conceptual foundation is not for all mathematics but for all algebraic structures, but it can be naturally extended and generalized to all mathematics in various ways (cf. Bourbaki, Category theory, Model Theory) including logic. That is the way to universal logic.

Like universal algebra, universal logic can be seen as a unifying theory: a theory unifying the *universe* of the multiplicity of logics, unification based on a conceptual basis rather than on an axiomatic one (cf. Beziau 2010b). A good title for a book

presenting a general study of the different systems of logic could be *The world of possible logics*—a title suggest to da Costa and me by Michel Paty, but we didn't write the book (yet). It is good because a *world* is something organized, at the same time it has an anthromorphic flavor. Another possibility would be to use the word *cosmos*, but maybe we would be too close to space opera while speaking of *Cosmology of Logics*. *Universal* is better because it catches the same idea through a semantic network encompassing both physics and mathematics: *universe* and *universal algebra*. *World* is too much down to the earth, and *cosmos* too much mythological.

As we have seen in the previous section, it is natural to consider that there are different logical systems even if we consider there is only one reality of reasoning. The variety of logical systems may reflect not only the various ways to understand reasoning at different stages of the history (syllogistic vs first-order logic) and various aspects (modalities, quantifiers, positive logic) of one real reasoning but also the variation of this one reasoning itself through different ages (children/adult) and different cultures. One may focus only on some aspects of the reality of reasoning, and then combine, compare, translate, etc., using general techniques developed to deal with logical structures.

On the other hand universal logic leaves open the possibility to consider that reasoning itself has some fundamental variations though history of humanities and also leaves open the logic door to the world: animals, plants, the universe …

3.3 Mathesis universalis and logica universalis

The semantic network of *universal* does not include only the *universe*, but also *university*, *universal gravitation*, *universal quantifier*, *universal declaration of human right*, *universal salvation*. This is a very powerful word, having religious, political, physical and logical dimensions, that should be used carefully. These four dimensions have developed together. The word *universal* comes from the French word *universel* coming itself from the latin word *universalis* which means "turned (*versus*) into one (*uni*)". What is central in *universal* is oneness. Something common to everything or that can be applied to everything.

Gian-Carlo Rota, a collaborator and friend of Garrett Birkhoff, wrote: "When we ask for foundations of mathematics, we must first look for the unstated wishes that motivate our questions. When you search into the Western mind, you discover the craving that all things should be reduced to one, that the laws of nature should all be the consequences of one law, that all principles should be reduced to one principle. It is a great Jewish idea. One God, one this, one that, one everything. We want foundations because we want oneness." (Rota 1997, p. 218). Oneness appeared in the Jewish culture through monotheism, contrasting with the polytheism of Greek and Roman religions. To conceive only one God is to be able to conceive unity in all the diversity of the world, a fundamental step in the development of humanity. After Jesus Christ, the oneness of the Jewish God was transformed in a religion for all, a universal religion. The Greek word "catholic" (*kata holos*) was used in this way, that is where the name "Catholic Church" came from. Christianism has also incorporated the universality of the Greek science identifying *logos* with God.

The universality of Christianism may have a normative character—everybody shall be converted to the true religion—connected with some normative rules derived from the ten commandments, which can be seen as some axioms. André Weil, the spiritual leader of Bourbaki was against that; he was more connected with Hindouism (the name *Bourbaki* was coined by him when he was in India). Weil wrote: "Claiming always to behave according to the precepts of universal maxims is either totally inept or totally hypocritical…. I have been deeply marked by Indian thought and by the spirit of the *Gita*…the law is not *Thou shalt not kill*, a precept that Judaism and Christianism have inscribed in their commandments…. In the absence of any universal recipe to prescribe to everyone's behavior, the individual carries within him his own *dharma*." (Weil 1991, p. 124). André Weil is the author of Bourbaki's famous paper "The architecture of mathematics", where he promoted a conceptual approach for the unity of mathematics rather than an axiomatic approach.

Another key figure of Bourbaki, Alexandre Grothendieck gave a lecture in 1970 on the occasion of Russell's centenary called "La nouvelle église universelle" (the new universal church) where he strongly criticizes scientism (Grothendieck 1974). He explains that religions have progressively disappeared and what is emerging is scientisim, a promotion of science having the defects of some religions: totalitarism (everybody should believe) and blindness (without properly understanding why); a paradox in view of the very nature of science.

Modern science didn't start in this way. We can consider that modern science started with Descartes's idea of *mathesis universalis*, whose main trait is understanding (Descartes's idea is to be found in Rule IV of the *Rules for the direction of mind*, Descartes 1628; for the origin of the expression, see Kauppi 1980). This approach is different from the later idea of Leibniz of a *characteristica universalis*, a universal language which, associated with a *calculus raciocinator,* would be a machine more powerful that human thought. As people have rightly pointed out, Leibniz was anticipating computation. (One of the best presentation of Leibniz's ideas on these topics is still Couturat 1901).

In the development of universal algebra, a journal was created with the name *Algebra Universalis* establishing explicitly the connection with *mathesis universalis* and *characteristica universalis*. In universal algebra there is the idea to develop some general and universal concepts that can be used and applied to the development of a huge variety of algebraic structures. It is more conceptual than linguistic, by contrast to some tendencies in formal logic (Peano, Frege, Russell,…)—the idea is not to construct a universal formal language. It is interesting to note that the abstract perspective of universal algebra at first looked very different from computation theory which was emerging at this same time but later on it served as a basis of the theory of computation (about that see Birkhoff 1987).

When creating a journal devoted to universal logic it was natural to call it *Logica Universalis*, going on with the analogy with universal algebra and the related journal *Algebra Universalis,* also published by Birkhäuser. We can say that *Logica Universalis* is in the line of *Mathesis Universalis, Characteristica Universalis, Algebra Universalis*. Among the four it seems that this is the one which makes more sense, if we remember that the Greek word *logos* means at the same time science, reasoning, language and relation.

4 Three roads to nowhere?

4.1 The fashion of logical pluralism

Since a couple of years, JC Beall and Greg Restall have promoted *logical pluralism*; it is an expression and a discussion related to it (the main book is Beall and Restall 2006). *Logical pluralism* can be simply understood as referring to a view according to which there are different logics: "To be a pluralist about logical consequence, you need only hold that there is more than *one true logic*." (Beall and Restall 2000, p. 476). But this expression is not innocent; it refers to a position supported by a gang, the logical pluralists, against another gang the logical monists. We are facing the "ism" mania like with communism, physicalism or mysticism.

Logical pluralism is the claim and the defense that there are various logics, it is not a general theory of logics. The logical pluralist does not make the distinction between reasoning and the theory of reasoning; both are put in the same bag. Beall and Restall use the distinction between "Logic" and "logic" in the following way: "*Logic* names the discipline, and *logic* names a logical system." (2000, p. 475). According to this view there is no clear distinction between a logic system and the reasoning it is describing. Furthermore logic, as a discipline does not explicitly appear as a systematic study of logical systems.

Beall and Restall write "A number of different formal logics, in particular, classical logics, relevant logics and intuitionistic logics, have their place in formalizing and regulating inference." (2000, p. 491). The emphasis is not on understanding the relation between the different logical systems by combining, comparing, translating, but to defend that each one has its legitimacy and that one is not necessarily better than the other one. Logical pluralism promotes plurality rather than unity. It is in the line with putting an "s" everywhere; physics, mathematics, religions, logics. The plural for physics or mathematics is indeed quite ambiguous. In English it is rather a syntactic feature. The idea of Bourbaki (1948) was to suppress the 's' in "mathématiques": the subtitle of their famous paper "The architecture of mathematics" is "La mathématique ou les mathématiques?"

The third criticism we can develop against logical pluralism is the atmosphere surrounding the expression "logical pluralism". In 2000 I was invited to a workshop on logical pluralism in Hobart, Tasmania, organized by JC Beall and Greg Restall with the participations of Francesco Paoli, Otávio Bueno and Achille Varzi. It was a friendly and relax meeting. At some point Greg said that "logical pluralism" was a sexy way of speaking. The adjective "sexy" has been popularized recently in the intellectual world, by a new generation of people not any more restrained by puritanism or moralism, who enjoy using this adjective: *sexy title*, *sexy talk*, *sexy expression*. Sexy is for them the symbol of attraction; this makes sense in fact from the point of view of the philosophy of Schopenhauer for whom sexuality was an aspect of universal gravitation (*The world as will and representation*, 1818).

"Logical pluralism" is linked in another way to sexuality: it is connected to homosexuality. The flag of homosexuality is the rainbow seen as a general symbol of pluralism opposed to the black and white dichotomy. It is a bit weird to promote plurality through a sexual activity between people of the same sex. It would be similar to promote

democracy through dictatorship saying that democrats are open to every politicians including dictators. However supporting homosexuality is politically correct.

To be pluralist is a politically correct way of being. The expression *politically correct* has progressively flourished during the last 30 years. It is now being used to characterize what is correct or not in the same sense than *morally correct* was used before. Moralism now looks quite old-fashion, but *politically correct* is just a new skin for the old ceremony. What is correct or not has changed but the *correctness mood* is the same: political correctness shares with the old-fashioned moralism the same blind normative aspect. One has to think or behave in a way without really understanding why and if one disobeys she (to use a politically correct way of speaking, contrasting somewhat with the sexism of using "sexy expressions") is considered as an eccentric or/and a dangerous female. And political correctness like the old moralism is full of absurdity and hypocrisy: for example, it is not politically correct to eat dogs; at the same time it is politically correct to eat cows; although it is politically correct to recognize the plurality of religions, the fact that for Hindus eating cows is not good.

Logical pluralism is fashionable and fashion is ephemeral and superficial, like a sexy young woman that 1 day will be a not so attractive old lady. To claim that logical pluralism is a fashionable nonsense would be more aggressive in the line of Sokal and Bricmont, and it is not necessary to go on up to this point. Nevertheless logical pluralism can be said to lack of meaning because it is not an articulated theory of logical systems.

4.2 The heresy of non-classical logics

Non-classical logics are quite popular nowadays. When did this stream start? The expression *non-classical logic* is opposed to *classical logic*, which is a system of logic developed mainly at three levels: classical propositional logic, classical first-order logic and classical second-order logic. When the adjective "classical" started to be used is difficult to specify. Classical propositional logic (CPC hereafter) is sometimes called Boolean logic, but the theory of Boole is quite different from CPC and, on the road from Boole to CPC, alternative logical systems appeared. Post (1921) who can be considered as the first to present the full-fledged CPC presented it together with the straightforward generalization of *many-valuedness*. We don't have a chronological development of modern logic with first CPC then non-classical logics. Many-valuedness symbolically appeared as multiple ways to develop logic in many theories of reasoning, based on the idea that there are many truth-values. Bivalent truth-functional logic is just one system among the multiplicity of many-valued truth-functional logics.

Intuitionistic logic also appeared quite simultaneously with classical logic. This makes sense if we remember that a strong direction in modern logic was to have a better understanding of mathematical reasoning. Intuitionistic logic was promoted by Brouwer as the right way to reason in mathematics and was later on displayed in a logical system, in particular by Heyting, but also by Gentzen (1934–1935) who developed simultaneously sequent systems for intuitionistic and classical logic. Funny enough Gödel (1932) has shown that intuitionistic logic cannot be characterized by a many-valued finite logical matrix. This result shows indeed that intuitionistic logic is

really different, it has also been shown by some translations methods that intuitionistic logic is not safer than classical logic (Gödel 1933). The absolutist has to be aware of the general situation.

From the point of view of intuitionistic logic, classical logic is wrong, the same idea has also been promoted by relevantists and all sort of constructivists, defending other alternative systems. The battle is tough: fuzzy logic has been compared to cocaine, paraconsistent logic to pornography…. Some non-classical logicians are not only against classical logic they are also against other logics thinking their logic is the best. Non-classical logic is not necessarily synonymous to pluralism.

It makes sense to say that *paraconsistent logic* is the climax of non-classical logic, because it is based on the rejection of what has been considered as one of the most traditional principles of logic since the beginning of logic: the principle of non-contradiction. Modern logic did not start with a rejection of it: Boole and Frege have completely transformed Aristotelian logic by proposing a new analysis of the proposition but they have not rejected the principle of non-contradiction. The first one to do this in the context of a logical theory is the Russian logician Nicolai Vasiliev from Kazan. By contrast to Boole and Frege, Vasiliev didn't propose a radical change in the understanding of the proposition but nevertheless he proposed a *non-Aristotelian* logic or *imaginary logic*, following a way of speaking analogous to Lobachevski also from Kazan—who launched the expressions *non-Euclidean geometry* and *imaginary geometry*. Vasiliev had little influence and the terminology did not last (see Bazhanov 1990). Later on was adopted the terminology *paraconsistent logic*, coined by Miró Quesada in a discussion with Newton da Costa (see da Costa et al. 1995). It is in fact more appropriate, because paraconsistent logic can be seen as an extension of classical logic. Paraconsistent logic is not necessarily opposed to classical logic (see Beziau 2000). A logic diametrically opposed to classical logic is *anti-classical logic* (see Beziau and Buchsbaum 2013).

Another important non-classical logic is *non-monotonic logic*. The negation here is not a negation of some laws concerning logical operators—connectives or quantifiers—but negation of some so-called structural principles (non-monotonic logic is considered as a typical *substructural logic*). According to monotonicity the validity of an argument is not modified by the addition of some hypotheses. This is a typical feature of mathematical reasoning and Tarski considered it as a basic axiom for a consequence relation. Tarski also considered two other axioms for consequence relations (reflexivity and transitivity). By rejecting these three axioms we are led to a logical structure in the the spirit of axiomatic emptiness of universal logic. This is not the direction taken by non-monotonists who are more concretely oriented. No flight of abstraction here, they are focusing on down to the earth problems: birds that cannot fly. Non-monotonic logic like many other non-classical logics is based on a modification of classical logic.

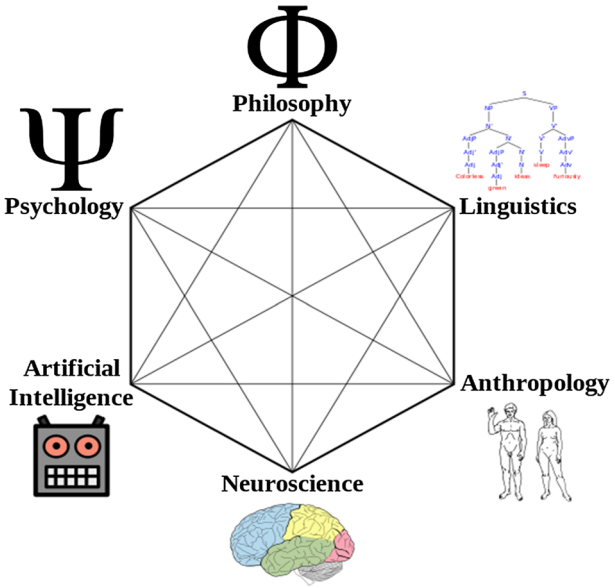
The non-classical logicians are in fact most of the time roaming around the house of classical logic, this is their reference point. Some want to burn the old house and build a brand new one. Others want to improve it, extending it with many operators: modalities, branching quantifiers, etc. Some guys just construct their own little house close by.

4.3 The mayonnaise of cognitive science

The advisor of my PhD thesis *Recherches sur la Logique Universelle* (1995) was Daniel Andler who did his PhD with Robert Vaught at Berkeley (1973). I met him in 1989 when he was teaching in the group of logic of the department of mathematics of the *Université Denis Diderot (Paris 7),* giving us a course on non-classical logics including, in particular, non-monotonic logics. This was considered as a kind of oddity in this group which had been developed by Kreisel in the spirit of foundations of mathematics and was slowly evolving towards foundations of computer science, lambda calculus taking the advantage over model and set theories. At this time this group was encompassing the main logicians working in France: Krivine, Girard, Lascar, etc., later on it was split in at least three groups due to the variety of personalities and lines of work.

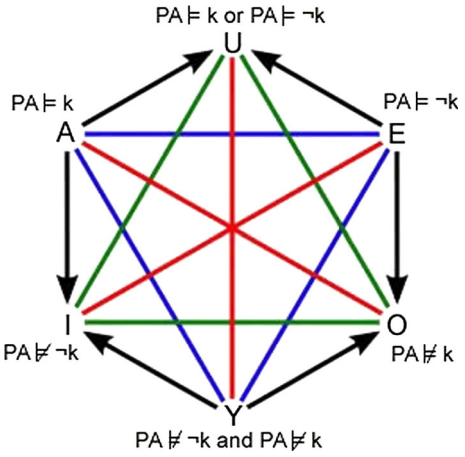
Andler wanted to develop new horizons and he worked hard to develop cognitive science and succeeded in creating a department of cognitive science at the *Ecole Normale Supérieure*. I think he did a great job and the criticisms of cognitive science presented here are not intended to underestimate his work. It is also worth pointing out that Andler supported my research on paraconsistent logic I did in my Master's thesis, encouraged me to go to Brazil to work with Newton da Costa and supported my project on universal logic. From him, I keep the positive view of cognitive science as a field with an open perspective.

But what is cognitive science exactly? We can roughly define it as the inquiry about how human mind works. A typical feature of cognitive science is that it is an interdisciplinary study of the mind, which is represented by the following hexagon (Miller 2003):



Interdisciplinarity is good but since the beginning it has encountered the problem of developing intelligently. It is not easy to go beyond the isolation of hyperspecialization, to construct something together. Cognitive science is maybe the highest tentative of interdisciplinarity and at least it succeeded in unifying around an idea, a word. But behind the word there is a lot of confusion/heterogeneity. Let us point out that the hexagon above reflects the superficiality of the interdisciplinarity of cognitive science: it is not a real structure, like the hexagon of opposition of Blanché (1966).

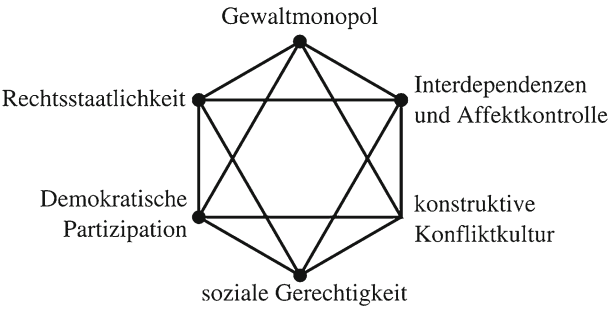
Blanché's hexagon articulates the relation between six notions using three notions of oppositions and the notion of subalternation. It is very useful to describe the interrelations of many concepts, like for example the six relations of a proposition vis-à-vis a theory in modern logic. This can be depicted by the following hexagon where PA is Peano Arithmetic and contradiction is in red, contrariety in blue, subcontrariety in green and subalternation in black:



The metalogical hexagon is interesting from the viewpoint of universal logic because it defines and describes the basic structure of the consequence relation of many different logics (the consequence relation appearing in this hexagon is not that of a specific logic and this metalogical hexagon can be generalized without negation, see Beziau 2012b).

On the other hand Blanché's hexagon is also a fundamental structure that can be applied to a wide variety of concepts outside of logic and it is possible to promote interdisciplinarity on the basis of this simple but rich structure as it has been recently done through the organization of world congresses and related publications (see Beziau and Payette 2012, Beziau and Jacquette 2012).

The cognitive science's hexagon looks more like many pseudo-hexagons which have in common with the hexagon of opposition just the form, for example the following hexagon of civilization (Zürn 2000):



Mixing the six topics linguistics, neuroscience, artificial Intelligence, philosophy, anthropology, and psychology, does not necessarily lead to a good mayonnaise. A kind of similar mixture already existed before, developed by the structuralist school. The new ingredients in cognitive science are neuroscience and artificial intelligence linking human sciences with biology and computers. That is fine but what is strangely missing in the cognitive hexagon is logic.

Why should logic, the theory of reasoning, be excluded from cognitive science? Is cognition not logical? And why including within the field of cognitive science some fields like psychology and anthropology, named with recent neologisms based on the word *logos* itself. *Anthropology* means the science of human beings; if human beings are defined as rational animals, should not anthropology be based on the theory of reasoning? It seems that this is not a real omission. It is an erasure. It rather looks like cognitive science wants to replace logic.

We can see cognitive science as the articulation between logic and neuroscience, a project I have been working on with Patrick Suppes at Stanford (2000–2001), but he didn't use the word cognitive science for it, he created a "brain" laboratory. Suppes grew up in the Tarskian School, he organized the congress on axiomatization of physics in the 1950s. When I was at Stanford he was finishing his book *Representation and Invariance of Scientific Structures* (2002), the result of 50 years of research.

In some sense the broad perspective of cognitive science is right but it lacks unity, a direction, some conceptual basis. At the beginning of the XXth century in Europe, people were using logic to develop the unity of science. Logic was identified to the methodology of deductive sciences (cf Tarski 1936 and the international series of events LMPS launched by Tarski). This project did not stop. Universal logic is the continuation of this project.

Acknowledgments Thanks to anonymous referees and all the people with whom I have been discussing these ideas over the years.

References

Bazhanov, V. A. (1990). The fate of one forgotten idea: N.A.Vasiliev and his imaginary logic. *Studies in Soviet Thought*, *39*(3–4), 333–334.

Beall, J. C., & Restall, G. (2000). Logical pluralism. *Australasian Journal of Philosophy*, *78*(4), 475–493.

Beall, J. C., & Restall, G. (2006). *Logical pluralism*. Oxford: Clarendon.

Beziau, J.-Y. (1994). Universal logic. In T. Childers & O. Majer (Eds.), *Logica'94–Proceedings of the 8th international symposium* (pp. 73–93). Prague.

Beziau, J.-Y. (1995). *Recherches sur la Logique Universelle*. PhD Thesis, Department of Mathematics, University Denis Diderot, Paris.
Beziau, J.-Y. (2000). What is paraconsistent logic? In D. Batens, et al. (Eds.), *Frontiers of paraconsistent logic* (pp. 95–111). Baldock: Research Studies Press.
Beziau, J.-Y. (2001). From paraconsistent to universal logic. *Sorites*, *12*, 5–32.
Beziau, J.-Y. (2006a). The paraconsistent logic Z—A possible solution to Jaskowski's problem. *Logic and Logical Philosophy*, *15*, 99–111.
Beziau, J.-Y. (2006b). 13 questions about universal logic. *Bulletin of the Section of Logic*, *35*, 133–150.
Beziau, J.-Y. (2010a). *Logic is not logic. Abstracta*, *6*, 73–102.
Beziau, J.-Y. (2010b). What is a logic? Towards axiomatic emptiness. *Logical Investigations*, *16*, 272–279.
Beziau, J.-Y. (Ed.). (2012a). Paralogics and the theory of valuation. In *Universal logic: An Anthology - From Paul Hertz to Dov Gabbay* (pp. 361–372). Birkhäuser, Basel.
Beziau, J.-Y. (2012b). The power of the hexagon. *Logica Universalis*, *6*, 1–43.
Beziau, J.-Y. (Ed.). (2012). *Universal logic, an anthology–From Paul Hertz to Dov Gabbay*. Basel: Birkhäuser.
Beziau, J.-Y., & Buchsbaum, A. (2013). Let us be antilogical: Anti-classical logic as a logic. In A. Moktefi, A. Moretti, & F. Schang (Eds.), *Let us be logical*. London: College Publications.
Beziau, J.-Y., & Jacquette, D. (2012). *Around and beyond the square of opposition*. Basel: Birkhäuser.
Beziau, J.-Y., & Payette, G. (2012). *The square of opposition—A general framework for cognition*. Bern: Peter Lang.
Birkhoff, G. (1946). Universal algebra. In Comptes Rendus (Ed.), *du Premier Congrès Canadien de Mathématiques* (pp. 310–326). Toronto: University of Toronto Press.
Birkhoff, G. (1987). Universal algebra. In G.-C. Rota & J. S. Oliveira (Eds.), *Selected papers on algebra and topology by Garret Birkhoff* (pp. 111–115). Basel: Birkhäuser.
Blanché, R. (1966). *Structures intellectuelles. Essai sur l'organisation systématique des concepts*. Paris: Vrin.
Bourbaki, N. (1948). L'architecture des mathématiques—La mathématique ou les mathématiques. In F. le Lionnais (Ed), *Les grands courants de la pensée mathématique, Cahier du Sud* (pp. 35–47); translated as "The Architecture of Mathematics", *American Mathematical Monthly, 57*, 221–232, 1950.
Carnap, R. (1934). *Logische Syntax der Sprache*. Vienna: Springer, translated in English as The logical syntax of language. London: Kegan Paul, 1937.
Couturat, L. (1901). *La Logique de Leibniz–D'après des documents inédits*. Paris: Félix Alcan.
da Costa, N. C. A., & Beziau, J.-Y. (1994a). Théorie de la valuation. *Logique et Analyse*, *145–146*, 95–117.
da Costa, N. C. A., & Beziau, J.-Y. (1994b). La théorie de la valuation en question. *Proceedings of the XI Latin American symposium on mathematical logic (Part 2)* (pp. 95–104). Bahia Blanca: Universidad Nacional del Sur.
da Costa, N. C. A., Beziau, J.-Y., & Bueno, O. A. S. (1995). Paraconsistent logic in a historical perspective. *Logique et Analyse*, *150–152*, 111–125.
Descartes, R. (1628). *Regulae ad directionem ingenii (Rules for the direction of mind)*, published in Amsterdam, 1701.
Descartes, R. (1637). *Discours de la méthode (Discourse on the method)*, Leyde.
Destousches, J.-L. (1948). *Cours de logique et philosophie générale*. Paris: Centre de document universitaire, Fournier & Constane.
Feferman, S., & Feferman, A. B. (2004). *Tarski: Life and logic*. Cambridge: Cambridge University Press.
Février, P. (1937). Les relations d'incertitude d'Heisenberg et la logique. In *Travaux du IXème Congrès International de Philosophie* (Vol. VI, pp. 88–94). Paris: Hermann.
Fitting, M. (1992). Preface. *Journal of Logic and Computation*, *2*(6), 783–785.
Gentzen, G. (1934–1935). Untersuchungen über das logische Schließen. *Mathematische Zeitschrift, 39*(2), 176–210, 39(3): 405–431.
Gödel, K. (1932). Zum intuitionistischen Aussagenkalkül. *Akademie der Wissenschaften in Wien, Mathematisch-natuirwissenschaft Klasse, 64*, 65–66.
Gödel, K. (1933). Zur intuitionistischen Arithmetik und Zahlentheorie. *Ergebnisse eines mathematischen Kolloquiums*, *4*, 34–38.
Gödel, K. (1949). An example of a new type of cosmological solution of Einstein's field equations of gravitation. *Review of Modern Physics*, *21*(3), 447–450.
Grothendieck, A. (1974). La nouvelle église universelle. *Pourquoi les mathmématiques* (pp. 11–35). Paris: UGE.

Henkin, L., Suppes, P. & A. Tarski (eds.). (1958). *The axiomatic method with special reference to geometry and physics. Proceedings of an international symposium held at the University of California, Berkeley, December 16, 1957–January 4, 1958* (pp. 291–307). Amsterdam: North-Holland.

Kauppi, R. (1980). Mathesis universalis. In J. Ritter & K. Gründer (Eds.), *Historisches Worterbuch der Philosophie* (Vol. 5, pp. 937–938). Basel and Stuttgart: Schwabe.

Marion, M. (2012). Louis Rougier on the relativity of logic—An early defence of logical pluralism. In Béziau.

Miller, G. A. (2003). The cognitive revolution: A historical perspective. *Trends in Cognitive Sciences*, *7*, 141–143.

Moschovakis, J. R. (2009). The logic of Brouwer and Heyting. In D. M. Gabbay & J. Woods (Eds.), *Handbook of the history of logic* (Vol. 5). Amsterdam: Elsevier.

Piaget, J. (1950). *Introduction à l'épistémologie génétique*. Paris: PUF. Translated as Genetic epistemology, Columbia University Press, New York, 1968.

Post, E. (1921). Introduction to a general theory of elementary propositions. *American Journal of Mathematics*, *13*, 163–185.

Riche, J. (2007). From universal algebra to universal logic. In J. Y. Beziau & A. Costa-Leite (Eds.), *Perspectives on universal logic* (pp. 3–39). Monza: Polimetrica.

Rota, G.-C. (1997). *Indiscrete thoughts*. Basel: Birkhäuser.

Rougier, L. (1941). The relativity of logic. *Philosophy and Phenomenological Research, 2*, 137–158, reprinted in (Béziau 2012).

Rougier, L. (1955). *Traité de la connaissance*. Paris: Gauthiers-Villars.

Rougier, L. (1977). *Le conflit du christianisme primitif et de la civilisation antique*. Paris: Copernic.

Sokal, A. (1996). A physicist experiments with cultural studies. *Lingua Franca*, 62–64.

Sokal, A., & Bricmont, J. (1997). *Impostures intellectuelles*. Paris: Odile Jacob. Translated as Fashionable nonsense, Picador, New York, 1998.

Suppes, P. (2002). *Representation and invariance of scientific structures*. Stanford: CSLI.

Suszko, R., &(with Brown, D.J.). (1973). Abstract logic. *Dissertationes Mathematicae*, *102*, 43–52.

Sylvester, J. J. (1884). Lectures on the principles of universal algebra. *American Journal of Mathematics*, *6*, 270–286.

Tarski, A. (1928). Remarques sur les notions fondamentales de la méthodologie des mathématiques. In Annales de la Société Polonaise de Mathématique) (Vol. 7, pp. 270–272), translated as "Remarks on Fundamental Concepts of the Methodology of Mathematics" In (Beziau ed 2012).

Tarski, A. (1936). *0 logice matematycznej i metodzie dedukcyjnej*, Ksiaznica-Atlas, Lwów and Warsaw (English translation; we refer here to the 4th edition by Jan Tarski: *Introduction to logic and to the methodology of the deductive sciences* (p. 1994). Oxford: OUP.

Tarski, A. (1937). Sur la méthode déductive. In *Travaux du IXe Congrès International de Philosophie* (Vol. VI, pp. 95–103). Paris: Hermann.

van Stigt, W. P. (1990). *Brouwer's intuitionism*. Amsterdam: North Holland.

Weil, A. (1991). *Souvenirs d'apprentissage*. Basel: Birkhäuser; translated as *The apprenticeship of a mathematician*, Basel: Birkhäuser.

Whitehead, A. N. (1898). *A treatise of universal algebra*. Cambridge: Cambridge University Press.

Whitehead, A. N., & Russell, B. (1910–1913). *Principia Mathematica*. Cambridge: Cambridge University Press.

Zürn, M. (2000). Vom Nationalstaat lernen, Das zivilisatorische Hexagon in der Weltinnenpolitik. In U. Menzel (Ed.), *University PressVom Ewigen Frieden und vom Wohlstand der Nationen* (pp. 21–25). Frankfurt.